A Lightweight Rao-Cauchy Detector for Additive Watermarking in the DWT-Domain

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Overview

1. Introduction
2. Distribution of DWT subband coefficients
3. Cauchy distribution
4. Rao hypothesis test
5. Results

Introduction

- Watermarking embeds a imperceptible yet detectable signal in multimedia content
- Blind watermarking detection does not have access to the unwatermarked host signal, thus host interferes with watermark detection
- Transform domains (DCT, DWT) facilitate perceptual and statistical modeling of the host
- Straightforward linear correlation detector only optimal for Gaussian host; DCT and DWT coefficient do not obey Gaussian law in general

Watermark Detection in Previous Work

- Using Likelihood ratio test (LRT)
  - host signal coefficients (DCT, DWT) modeled by GGD [Hernández et al., 2000]
  - host signal coefficients (DCT) modeled by Cauchy distribution [Briassouli et al., 2005]
  - LRT is optimal, but assumes that watermark power is known
- Using Rao test
  - GGD host model [Nikolaidis and Pitas, 2003]
  - Rao test makes no assumption on watermark power, but is only asymptotically equivalent to the GLRT
  - GGD parameter estimation is computationally expensive

Distribution of DWT detail subband coefficients

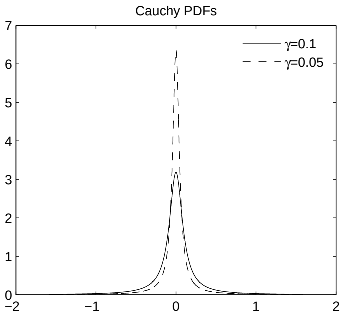
- GGD model known to fit DCT AC and DWT detail subband coefficients
- GGD parameters expensive to compute
- Often set GGD shape parameter to fixed value (eg. 0.5 or 0.8 for DCT/DWT coefficients)
- Alternative: Cauchy distribution

Cauchy Distribution

- Cauchy has been applied to blind DCT-domain spread-spectrum watermarking [Briassouli et al., 2005]
- Cauchy distribution PDF

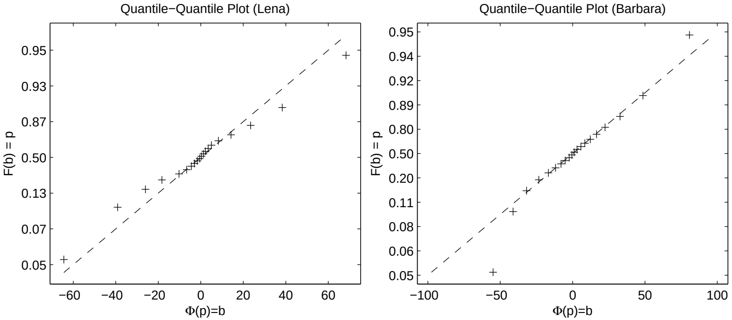
$$p(x|\gamma, \delta) = \frac{1}{\pi} \frac{\gamma}{\gamma^2 + (x - \delta)^2},$$

with location parameter $-\infty < \delta < \infty$ and shape parameter $\gamma > 0$



Q-Q Plots of DWT Detail Subband Coefficients

Decomposition level 2, horizontal orientation (H_2 subband)



Detection Problem

- Consider DWT detail subband coefficients as i.i.d. random variables following a Cauchy distribution with parameters γ and $\delta = 0$
- Want to detect deterministic signal of unknown amplitude (the watermark scaled by strength parameter α) in Cauchy distributed noise (the host signal)

$$\mathcal{H}_0 : \alpha = 0, \gamma \text{ (no/other watermark)}$$
$$\mathcal{H}_1 : \alpha \neq 0, \gamma \text{ (watermarked)}$$

Rao Hypothesis Test

- Two-sided composite hypothesis testing problem with one nuisance parameter γ
- In contrast to GLRT, Rao test does not require to estimate unknown parameter α under $\mathcal{H}_1$
- For symmetric PDFs [Kay, 1989], the Rao test statistic for our watermark detection problem can be written as

$$\rho(\mathbf{y}) = \left[\sum_{i=1}^{N} \frac{\partial \log p(y[i] - \alpha w[i], \hat{\gamma})}{\partial \alpha} \bigg|_{\alpha=0} \right]^2 \mathbf{I}_{\alpha\alpha}^{-1}(0, \hat{\gamma})$$

$p(\cdot)$ denotes the Cauchy PDF, $\hat{\gamma}$ is the MLE of the Cauchy shape parameter, $\mathbf{I}_{\alpha\alpha}^{-1}$ is an element of the Fisher Information matrix

Detection Statistic

After simplifications (inserting the Cauchy PDF and determining $\mathbf{I}^{-1}_{\alpha\alpha}(0, \hat{\gamma})$), the detection statistic becomes

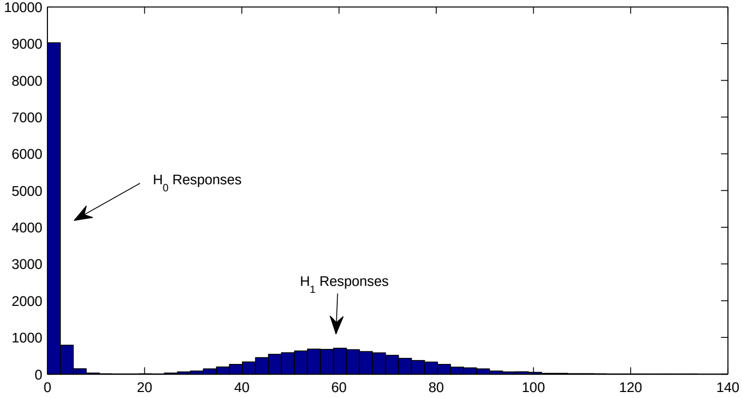
$$\rho(\mathbf{y}) = \left[\sum_{t=1}^{N} \frac{y[t]w[t]}{\hat{\gamma}^2 + y[t]^2} \right]^2 \frac{8\hat{\gamma}^2}{N}$$

with the asymptotic property

$$\rho \overset{a}{\sim} \begin{cases} \chi^2_1, & \text{under } \mathcal{H}_0 \\ \chi^2_{1,\lambda}, & \text{under } \mathcal{H}_1 \end{cases}$$

$\chi^2_{1,\lambda}$ denotes the non-central χ^2 distribution with non-centrality parameter λ

Detection Responses under $\mathcal{H}_0$ and $\mathcal{H}_1$



Detection Probability

- Since the distribution of the detector response ρ under $\mathcal{H}_0$ and $\mathcal{H}_1$ is known, we can express the probability of false-alarm (P_f), detection (P_d) and miss (P_m) as

$$P_f = \mathbb{P}\{\rho > T|\mathcal{H}_0\} = \mathrm{Q}_{\chi_1^2}(T) = 2\,\mathrm{Q}(\sqrt{T})$$

$$P_m = 1 - P_d = 1 - \mathbb{P}(\rho > T|\mathcal{H}_1) = 1 - \mathrm{Q}(\sqrt{T} - \sqrt{\lambda}) + \mathrm{Q}(\sqrt{T} + \sqrt{\lambda})$$

  where T denotes the detection threshold and Q is used to express right-tail probabilities of the Gaussian distribution.

- The ROC can be plotted using

$$P_m = 1 - \mathrm{Q}(\mathrm{Q}^{-1}(P_f/2) - \sqrt{\lambda}) - \mathrm{Q}(\mathrm{Q}^{-1}(P_f/2) + \sqrt{\lambda})$$

  where we have expressed P_m depending on P_f.

Host Signal Parameter Estimation

To determine the MLEs for the Cauchy or GGD shape parameter, we have to solve

$$\frac{1}{N}\sum_{t=1}^{N}\frac{2}{1+(x[t]/\hat{\gamma})^2}-1=0 \quad \text{(Cauchy)}$$

or

$$1+\frac{\psi(1/\hat{c})+\log\left(\frac{\hat{c}}{N}\sum_{t=1}^{N}|x[t]|^{\hat{c}}\right)}{\hat{c}} - \frac{\sum_{t=1}^{N}|x[t]|^{\hat{c}}\log(|x[t]|)}{\sum_{t=1}^{N}|x[t]|^{\hat{c}}}=0 \quad \text{(GGD)}$$

numerically. Approximately the same number of iterations are necessary (Newton-Raphson), however the computation effort is much higher for the GGD.

Detector Comparison: Computational Effort

Number of arithmetic operations to compute detection statistic for signal of length N

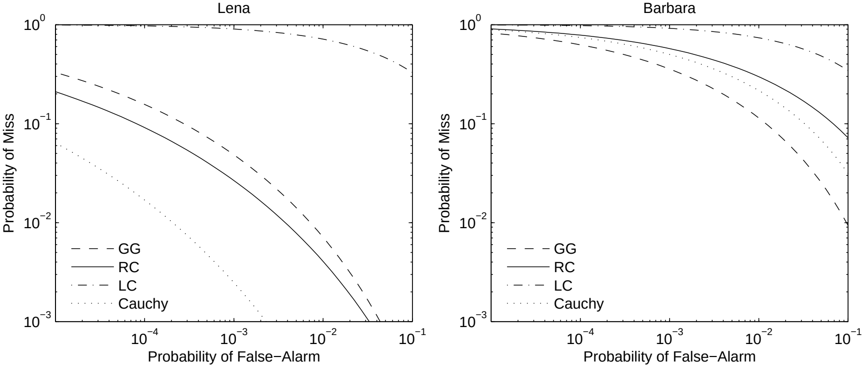
Detector	Operations			
	+,-	$\times, \div$	pow, log	abs, sgn
LC	N	N		
Rao-Cauchy	2N	2N+4		
Rao-GGD [Nikolaidis and Pitas, 2003]	2N	3N+1	2N	3N
LRT-GGD [Hernández et al., 2000]	3N	2	2N+1	2N
LRT-Cauchy [Briassouli et al., 2005]	4N	5N	N	

Rao-Cauchy Detector: Advantages / Disadvantages

+ Easier parameter estimation for Cauchy distribution over GGD
+ Rao detection statistic requires less computational effort than LRT
+ No unknown parameters in the asymptotic PDF under $\mathcal{H}_0$ (constant false-alarm rate detector)
+ No knowledge of embedding strength required for computation of detection statistic
− Rao test only asymptotically equivalent to GLRT (no optimality associated with GLRT)
− Cauchy is a rough approximation of DWT detail subband statistics, especially in the tail regions (too heavy)

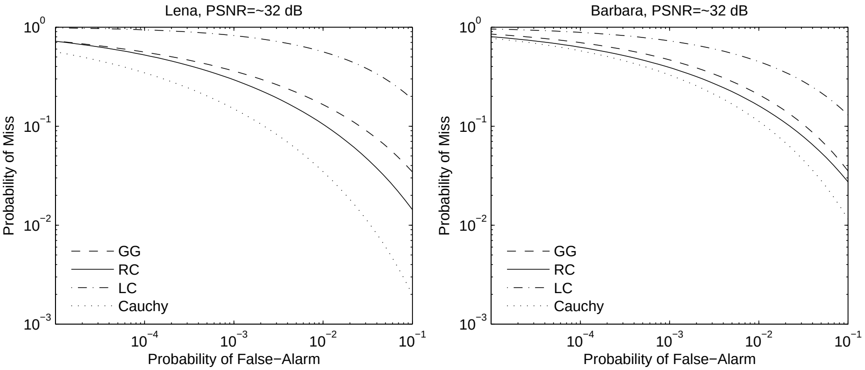
Detection Performance: Experimental Results

Embedding with 25 dB DWR



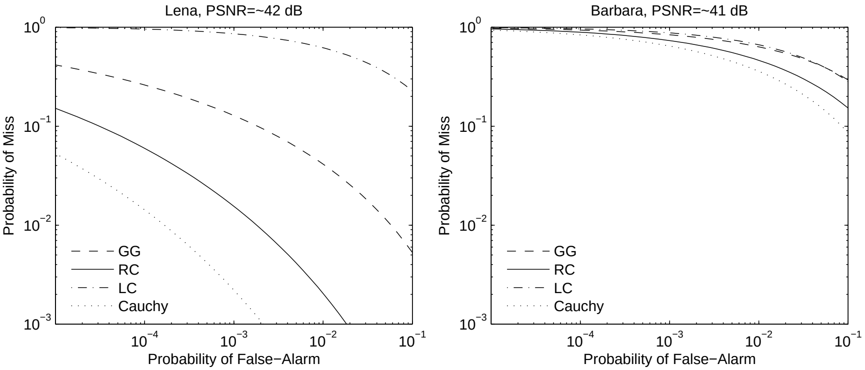
JPEG Compression Attack

JPEG compression, Q=50; embedding DWR 20 dB



JPEG2000 Compression Attack

Jasper JPEG2000 codec, 2.4 bpp; embedding DWR 23 dB



Conclusion

- DWT detail subband coefficients can be modeled by one-parameter Cauchy distribution
- Proposed Rao hypothesis test for Cauchy host data
- Parameter estimation of the Cauchy distribution is less expensive than for the GGD
- Computation of detection statistic for the Rao-Cauchy test more efficient than the LRT conditioned to the GGD or Cauchy distribution
- Rao-Cauchy detector has competitive detection performance
- Source code available on request: `http://wavelab.at/sources`

References

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